647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2024 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three. (10)

1. Let X is a discrete metric space and $a \in$ X. Show that for
$$N\ (a, \delta) = \begin{cases} \{a\}, & 0 < \delta \le 1 \\ X, & \delta > 1 \end{cases}$$
2. Let X is a metric space. Show that intersection of finite number of open subsets of X is open. Explain with an example that this statement is not true for infinite number of open subsets of X.
3. Explain cantor set and define it. Show that cantor set is closed.

Que.1(B) Answer any one out of two. (04)

1. Show that neighborhood is open set in metric space.
2. (i) If E = [a,b] then find E^0 in usual notation.

   (ii) Check whether the set E = $\left\{ \frac{1}{n} / n \in \mathbb{N} \right\}$ is open, close, both or none.

Que.2(A) Answer any two out of three. (10)

1. If bounded function *f* on [a,b] is decreasing then show that *f* is R-integrable in [a,b].
2. If *f* is R-integrable on [a,b] then for all $\lambda \in \mathbb{R}$ show that λf is also R-integrable on [a,b].
3. Let function *f* is bounded in [a,b] and P and P^* be two partitions of [a,b] such that $P \subset P^*$, then show that $U(P^*, f) \le U(P, f)$.

Que.2(B) Answer any one out of two. (04)

1. If $f(x) = x^3$, $x \in [a,b]$ where a>0 then show that $f \in R_{[a,b]}$ and also find $\int_0^a f(x)dx$.
2. If function *f* is continuous on [a,b] then prove that $f \in R_{[a,b]}$.

Que.3(A) Answer any two out of three. (10)

1. Using Darboux theorem find value of
$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots\ldots\ldots\ldots + \left(\frac{n}{n}\right)^2\right]$$
2. State and prove general form of first mean value theorem for R-integration.
3. Define a group and show that it satisfies cancellation laws.

Que.3(B) Answer any one out of two. (04)

1. Show that $\int_0^{\frac{\pi}{4}} x\ tan\ (x)\ dx < \frac{\pi}{8} \log_e 2$.
2. In usual notation, prove that $(\mathbb{Z}_4, +_4)$ is a group. Find order of each of it's elements.

Que.4(A) Answer any two out of three. (10)

1. State and prove Lagrange's theorem.
2. Prove that every permutation of a finite set is a product of disjoint cycles.
3. Let G be a finite group. For all a,b $\in$ G show that o($b^{-1}ab$) = o(a).

Que.4(B) Answer any one out of two. (04)

1. Let G be a group and H be it's subgroup. Prove that number of elements in every right coset of H in G is equal to o(H).
2. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 5 & 4 & 2 & 1 \end{pmatrix} \in S_6$ then find σ^{-1} and o(σ). Also check whether σ is even or odd.

Que.5(A) Answer any two out of three. (10)

1. State and prove Cayley's theorem.
2. Prove that for finite cyclic group, order of group is same as order of it's generator.
3. Prove that relation "isomorphism" between two groups is an equivalence relation.

Que.5(B) Answer any one out of two. (04)

1. Let G be a group and H be it's subgroup. If index of H in G is 2 then prove that H is normal subgroup of G.
2. Prepare group table in usual notations, for quotient group S_3/A_3.

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